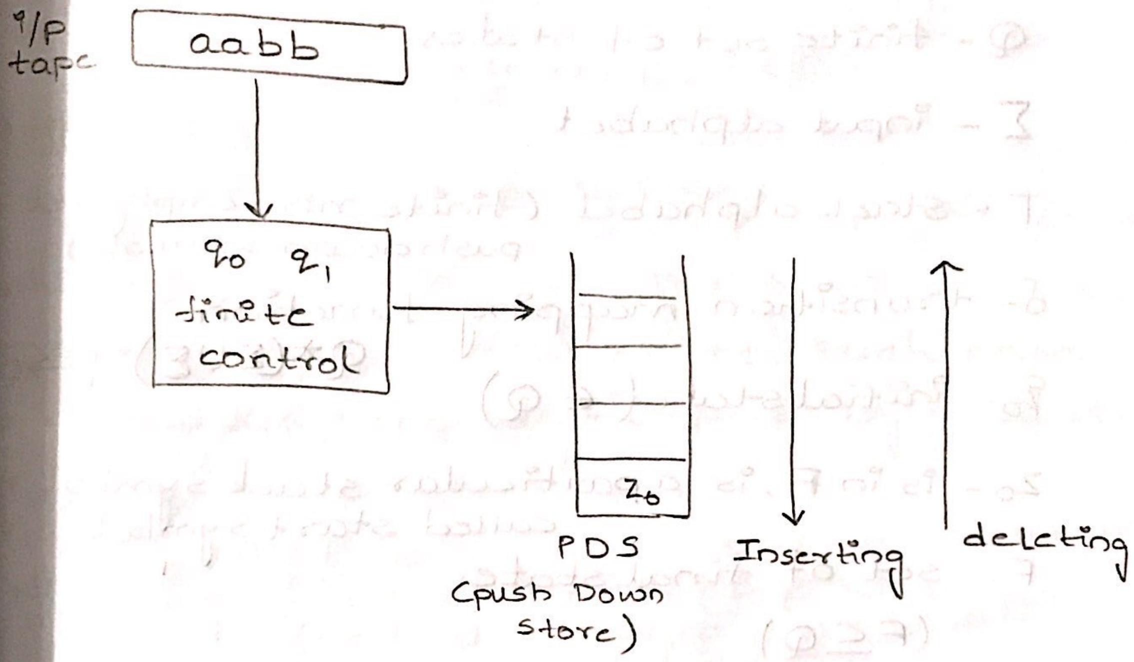
4. PUSH DOWN AUTOMATON

Push down automaton:



→ It has read only i/p tape, i/p alphabet, finite state control, a set of final states and an initial state. It has a stack called the push downstore (PDS).

→ We can add elements (or) remove elements from PDS. The PDA some state and reading an i/p symbol, top most symbol in PDS moves to new state & add string of symbols in PDS.

→ Left most symbol of stack is considered as top of the stack.

$$\boxed{M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)}$$

A PDA, M is a 7 tuple given as

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ where

Q - finite set of states

Σ - input alphabet

Γ - stack alphabet (finite non-empty set of pushdown symbols)

δ - transition mapping function $Q \times (\Sigma \cup \varepsilon)^* \Gamma \to Q \times \Gamma^*$

q_0 - initial state ($q_0 \in Q$)

Z_0 - is in F, is a particular stack symbol called start symbol

F - set of final state.

($F \subseteq Q$)

$Q \times (\Sigma \cup \varepsilon) \times \Gamma \to Q \times \Gamma^*$

$\delta(q, a, x) \to (p, \gamma)$

p - new state.

γ - string of stack symbol.

if $\gamma = \varepsilon$, then stack is pop

$\gamma = x$, then stack is not changed

$\gamma = yz$, then x is replaced by z and y is pushed down to stack.

Language accepted by PDA:

→ Two types

a) Empty / NULL stack:

Set of all i/p's for which some sequence of moves causes the PDA to empty its stack, in such case we say that language accepted by empty/NULL stack given as

$N(M) = \{ w \mid (q_0, w, Z_0) \vdash^* (p, \epsilon, \epsilon) \}$ ($p \in Q$)

turnstyle

(instantaneous description)

b) final state:

Similar to F.A. i.e., set of all inputs for which some choice of moves causes the PDA to enter a final state, such case is called as language accepted by final state given as

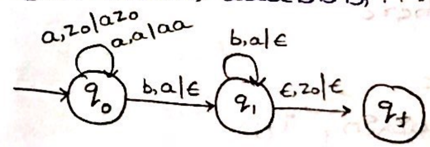
$N(M) = \{ w \mid (q_0, w, Z_0) \vdash^* (p, \epsilon, \gamma) \}$ ($p \in Q$, $\gamma \in \Gamma^*$)

F

Design PDA for the lang accepting $L = \{a^n b^n \mid n \geq 1\}$ by empty stack.

$L = \{ab, aabb, aaabbb, \ldots\}$

w = aabb



$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \varepsilon)$

$\delta(q_1, b, a) = (q_1, \varepsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_f, \varepsilon)$ - empty stack.

$(q_f, Z_0) \to$ final state

∴ Lang accepted by PDA (Empty stack)

→ $M(\{q_0, q_1, q_f\}, \{a, b\}, \{a, b, Z_0\}, \delta, q_0, Z_0, q_f)$

$w = aabb$

$\delta(q_0, aabb, z_0) \vdash (q_0, abb, az_0)$

$\vdash (q_0, bb, aaz_0)$

$\vdash (q_1, b, az_0)$

$\vdash (q_1, \epsilon, z_0)$

$\vdash (q_1, \epsilon, \epsilon)$

$\Downarrow$

string accepted by empty stack

<u>Instantanous description of a PDA:</u>

The PDA computes based on the state, i/p symbol and content of stack.

Each computation is called as instantanous description (<u>ID</u>)

→ The configuration of PDA is given by a <u>triple (q, w, γ)</u> where
ID

q - state

w - remaining i/p

γ - stack content.

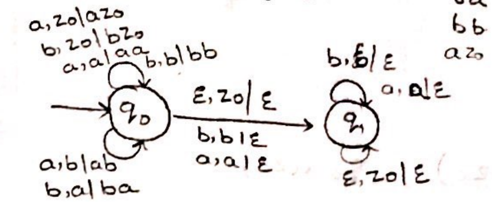
→ For connecting the pair of ID's we use a symbol called "<u>turn style</u>" denoted as "$\vdash$" represents one or more moves of a PDA. Suppose $\delta(q, a, x)$ has an o/p (P, γ) then for all strings w in Σ^* and β in Γ^* we have

$\delta(q, aw, x\beta) \vdash (P, w, \gamma\beta)$

2) Construct PDA to accept a set of all pallendromes over $\{a,b\}$ by nullstore / design a PDA to accept

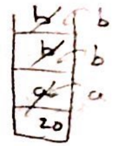
$L = \{ww^R \mid w \in \{a,b\}^*\}$

$w = abb$
$w = aab$
$w = baa$
$w = bba$



bz0
aa
ab
ba
bb
az0

$w = abbbba$



$\delta(q_0, a, z_0) = (q_0, az_0)$ $\quad$ $\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$ $\quad$ $\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, \varepsilon, z_0) = (q_1, \varepsilon)$

$\delta(q_0, b, b) = (q_1, \varepsilon)$

$\delta(q_0, a, a) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)$

$\delta(q_1, a, a) = (q_1, \varepsilon)$

$\delta(q_1, b, b) = (q_1, \varepsilon)$

$w = abb|bba$

$\delta(q_0, abbbba, z_0) \vdash (q_0, bbbba, az_0)$

$\vdash (q_0, bbba, baz_0)$

$\vdash (q_0, bba, \overset{bb}{a}z_0)$

$\vdash (q_\phi, ba, baz_0)$

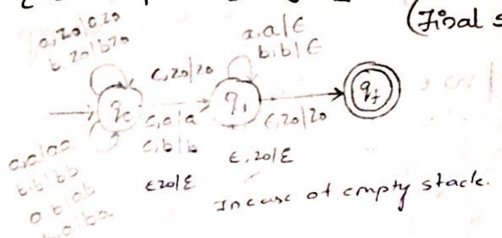
$\vdash (q_\phi, a, az_0)$

$\vdash (q_\phi, \varepsilon, z_0)$ $\quad$ $\vdash (q_1, \varepsilon, \varepsilon)$

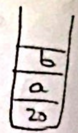
Empty stack.

3) $L = \{wcw^R \mid w \in \{a,b\}^*\}$

(Final state)



ω = abcba



$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, c, z_0) = (q_1, z_0)$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_f, z_0)$

Acceptence of string.

ω = abc~~ab~~ba

$\delta(q_0, abcab, z_0) \vdash (q_0, bc\overset{ba}{\cancel{ab}}, az_0)$

$\vdash (q_0, c\overset{ba}{\cancel{ab}}, baz_0)$

$\vdash (q_1, \overset{ba}{\cancel{ab}}, baz_0)$

$\vdash (q_1, a, az_0)$

$\vdash (q_1, \epsilon, z_0)$

$\vdash (q_f, \epsilon, z_0) \rightarrow$ Accepted by final state.

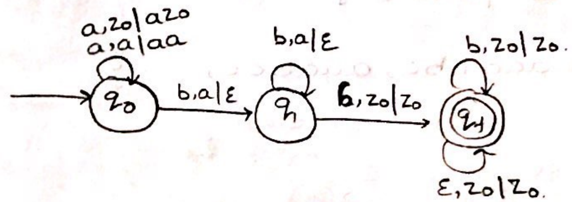
4) $L = \{a^n b^m \mid \overset{m \geq n}{\cancel{n \geq m}}, \overset{n}{\cancel{m}} \geq 1\}$

$L = \{ab, abb, aabb, \ldots\}$.

No. of b's > no. of a's.

{ cancel a's with b's. }
{ cancel b's with b's. }

⟹

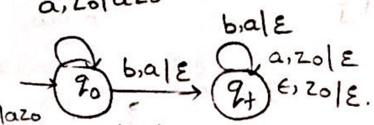


(correct)

5) $L = \{a^n b^m \mid n \geq m, m \geq 1\}$

a,a|aa
a,z0|az0

⟹



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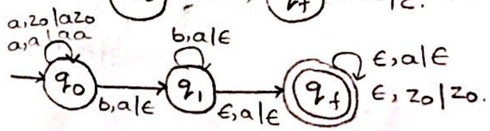
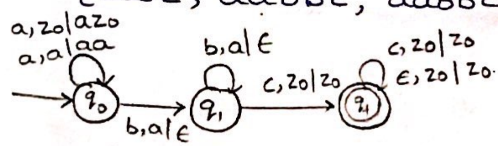


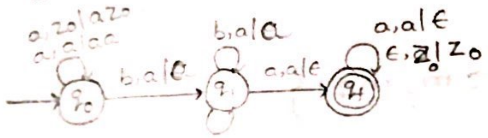
6) $L = \{a^n b^n c^m \mid n, m \geq 1\}$

$L = \{abc, aabbc, aabbcc, \ldots\}$



7) $L = \{a^n b^m a^n \mid n, m \geq 1\}$

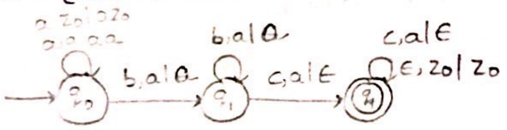
$L = \{aba, aabaa, \ldots\}$



aba
a

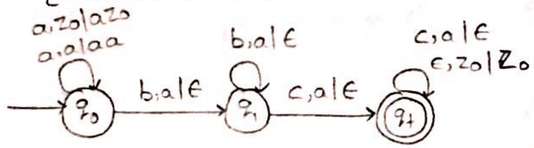
8) $L = \{a^n b^m c^n \mid n, m \geq 1\}$

$L = \{abc, aabcc, abbc, \ldots\}$



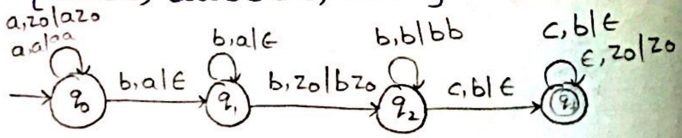
9) $L = \{a^{m+n} b^m c^n \mid m, n \geq 1\}$

$L = \{aabc, aaabbc, aaabcc, \ldots\}$



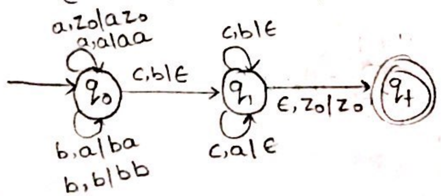
10) $L = \{a^n b^{m+n} c^m \mid m, n \geq 1\}$

$L = \{abbc, aabbbc, \ldots\}$



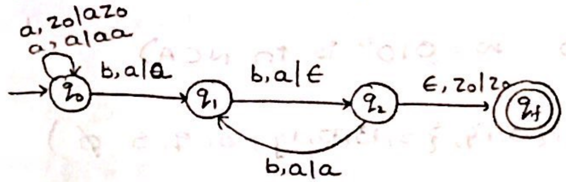
11) $L = \{a^n b^m c^{m+n} \mid m, n \geq 1\}$

$L = \{abcc, aabccc, abbccc, \ldots\}$

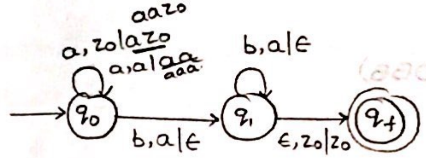


12) $L = \{a^n b^{2n} \mid n \geq 1\}$

$L = \{abb, aabbbb, \ldots\}$



(or)



?

⇒ construction of PDA, equivalent to the given CFL/CFG.

If L is a context free language then we can construct a PDA M accepting L by empty stack ie, $L = N(M)$

Let G - grammar representing L, we construct a PDA M by making use of productions in G.

(construction of M)

Step 1: $L = L(G)$ where $G = (V, T, P, S)$ is a CFG. we construct a PDA M as

$$M = (\{q\}, T, V \cup T, \delta, q, S, \phi)$$

empty stack

where δ is defined by following rules

R_1 : $\delta(q, \epsilon, \not{\phi}, A) = \{(q, \alpha) / A \rightarrow \alpha$ is in P)

R_2 : $\delta(q, a, a) = \{(q, \epsilon)$ for every a in T)

Step 2:

Proof of construction.

Take any $w \in L(G)$ and check acceptance of w.

1) construct a PDA equivalent to grammar

$S \rightarrow 0BB\cancel{B}$

$B \rightarrow 0S|1S|0$ $w = 010^4$ is in $N(A)$.

$M = (\{q\}, \{0,1\}, \{S, B, 0, 1\}, \delta, q, S, \phi)$

$S \rightarrow 0BB$

R_1: $\delta(q, \epsilon, S) = (q, 0BB)$

$B \rightarrow 0S|1S|0$

R_1: $\delta(q, \epsilon, B) = \{(q, 0S), (q, 1S), (q, 0)\}$

R_2: $\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

$S \Rightarrow 0BB$

$\Rightarrow 01\underline{S}B$

$\Rightarrow 010BBB$

$\Rightarrow 010000$

Given $w = 010000$

$\delta(q, 010000, \underline{S}) \vdash (q, \underline{0}10000, \underline{0}BB)$

$\vdash (q, \epsilon 10000, \underline{B}B)$

$\vdash (q, \underline{1}0000, \underline{1}SB)$

$\vdash (q, \epsilon 0000, \underline{S}B)$

$\vdash (q, 0000, 0BBB)$

$\vdash (q, \epsilon 000, \underline{B}BB)$

$\vdash (q, 000, 0BB)$

$\vdash (q, \epsilon 00, BB)$

$\vdash (q, 00, 0B)$

$\vdash (q, \epsilon 0, \underline{B})$

$\vdash (q, 0, 0)$

$\vdash (q, \epsilon, \epsilon)$

2) construct a PDA for accepting lang $L = \{a^n b^n \mid n \geq 1\}$ by null stack.

$L = \{a^n b^n\}$

$S \rightarrow aSb|ab$.

R_1: $\delta(q, \epsilon, S) = \{(q, aSb), (q, ab)\}$

R_2: $\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$S \Rightarrow aSb$

$\Rightarrow aabb$

Let, $w = aabb$.

$\delta(q, \epsilon aabb, \underline{S}) \vdash (q, aabb, aSb)$

$\vdash (q, \epsilon abb, Sb)$

$\vdash (q, abb, abb)$

$\vdash (q, bb, bb)$

$\vdash (q, b, b)$

$\vdash (q, \epsilon, \epsilon)$

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3) Construct PDA for the following CFG

$S \rightarrow 0S1 \mid A$

$A \rightarrow 1A0 \mid S \mid \epsilon$, $\omega = 001011$

$S \Rightarrow 0S1$
$\Rightarrow 00S11$
$\Rightarrow 00A11$
$\Rightarrow 001A011$
$\Rightarrow 001011$

$\delta(q,\epsilon,S) = \{(q,0S1),(q,A)\}$

$\delta(q,\epsilon,A) = \{(q,1A0),(q,S),(q,\epsilon)\}$

$\delta(q,0,0) = (q,\epsilon)$

$\delta(q,1,1) = (q,\epsilon)$

Given $\omega = 001011$

$\delta(q,\epsilon 001011,S) \vdash (q, 001011, 0S1)$

$\vdash (q,\epsilon 01011, S1)$

$\vdash (q, 01011, 0S11)$

$\vdash (q,\epsilon 1011, S11)$

$\vdash (q,\epsilon 1011, A11)$

$\vdash (q, 1011, 1A011)$

$\vdash (q,\epsilon 011, A011)$

$\vdash (q, 011, 011)$

$\vdash (q,11,11)$

$\vdash (q,1,1)$

$\vdash (q,\epsilon,\epsilon)$

4) Construct PDA for following CFG

$S \rightarrow SS \mid [S] \mid \{S\} \mid \epsilon$

$\omega = \{[\,]\}$

$\omega = \{[\,]\}[\,]$

1) $S \Rightarrow \{S\}$
$S \Rightarrow \{[S]\}$
$S \Rightarrow \{[\,]\}$

2) $S \Rightarrow SS$
$\Rightarrow \{S\}S$
$\Rightarrow \{[S]\}S$
$\Rightarrow \{[\,]\}S$
$\Rightarrow \{[\,]\}[S]$
$\Rightarrow \{[\,]\}[\,]$

$\delta(q,\epsilon,S) = \{(q,SS),(q,[S]),$
$(q,\{S\}),(q,\epsilon)\}$

$\delta(q,\{,\{) = (q,\epsilon)$

$\delta(q,\},\}) = (q,\epsilon)$

$\delta(q,[,[) = (q,\epsilon)$

$\delta(q,],]) = (q,\epsilon)$

(i) $\omega = \{[\,]\}$

$\delta(q,\epsilon\{[\,]\},S) \vdash (q,\{[\,]\},\{S\})$

$\vdash (q,\epsilon[\,]\},S\})$

$\vdash (q,[\,]\},[S]\})$

$\vdash (q,\epsilon]\},S]\})$

$\vdash (q,]\},]\})$

$\vdash (q,\},\})$

$\vdash (q,\epsilon,\epsilon)$

(ii) $\omega = \{[\,]\}[\,]$

$\delta(q, \epsilon\{[\,]\}[\,], S) \vdash (q, \epsilon\{[\,]\}[\,], SS)$

$\vdash (q, \{[\,]\}[\,], \{S\}S)$

$\vdash (q, [\,]\}[\,], S\}S)$

$\vdash (q, [\,]\}[\,], [S]\}S)$

$\vdash (q, \epsilon]\}[\,], S]\}S)$

$\vdash (q,]\}[\,],]\}S)$

$\vdash (q, \}[\,], \}S)$

$\vdash (q, \epsilon[\,], S)$

$\vdash (q, [\,], [S])$

$\vdash (q, \epsilon], S])$

$\vdash (q,],])$

$\vdash (q, \epsilon, \epsilon)$

5) $S \to aB \mid bA$

$A \to aS \mid a \mid bAA$

$B \to bS \mid b \mid aBB$

$\delta(q, \epsilon, S) = \{(q, aB), (q, bA)\}$

$\delta(q, \epsilon, A) = \{(q, aS), (q, a), (q, bAA)\}$

$\delta(q, \epsilon, B) = \{(q, bS), (q, b), (q, aBB)\}$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\omega = abba$

$S \Rightarrow aB$
$\Rightarrow abS$
$\Rightarrow abbA$
$\Rightarrow abba$

$\delta(q, \epsilon abba, S) \vdash (\delta, abba, aB)$

$\vdash (\delta, \epsilon bba, B)$

$\vdash (\delta, bba, bS)$

$\vdash (\delta, \epsilon ba, S)$

$\vdash (\delta, ba, bA)$

$\vdash (\delta, \epsilon a, A)$

$\vdash (\delta, a, a)$

$\vdash (\delta, \epsilon, \epsilon)$

6) construct PDA for accepting odd & even palindromes over $\{a,b\}$ by Null store.

(i) $\{\omega\omega^R\}$ (ii) $\{\omega c \omega^R\}$

$L = \{ww^R\}$

$S \to aSa \mid bSb \mid \varepsilon$

$\delta(q, \epsilon, S) = \{(q, aSa), (q, bSb), (q, \varepsilon)\}$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\omega = aa$

$\delta(q, \epsilon aa, S) \vdash (q, \cancel{aSa}\, aa, aSa)$

$\vdash (q, \epsilon a, Sa)$

$\vdash (q, a, a)$

$\vdash (q, \epsilon, \epsilon)$

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(ii) $L = \{wcw^R\}$

$S \to aSa \mid bSb \mid a \mid b$

$\delta(q, \epsilon, S) = \{(q, aSa), (q, bSb), (q, a), (q, b)\}$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$w = aba$

$\delta(q, aba, S) \vdash (q, aba, aSa)$

$\vdash (q, \epsilon ba, Sa)$

$\vdash (q, ba, ba)$

$\vdash (q, a, a)$

$\vdash (q, \epsilon, \epsilon)$

Conversion of PDA to grammar (CFG):

Let M be the PDA $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, \phi)$, $G = (V, T, P, S)$ be a CFG where

V – set of objects of form $[q, A, P]$, q & P in Q and A in Γ and new symbol "S"

P – set of productions.

① $S \to [q_0, Z_0, q]$ for each q in Q $q_0, q_1, \ldots$

② $[q, A, q_{m+1}] \to a[q_1, B_1, q_2][q_2, B_2, q_3] \ldots [q_m B_m q_{m+1}]$

according to problem.

→ $q_1, q_2, \ldots q_{m+1}$ in Q

→ each a in $\Sigma \cup \{\epsilon\}$

→ $A, B_1, B_2 \ldots B_m$ in Γ

$\delta(q, a, A) = (q_1, B_1, B_2 \ldots B_m)$

③ If $m = 0$ then the production is

$[q, A, q_1] \to a$

If $\delta(q, a, A) = (q_1, \epsilon)$

1) Let $m = (\{q_0, q_1\}, \{0, 1\}, \{x, Z_0\}, \delta, q_0, \underline{Z_0}, \phi)$

δ is given by $\delta(q_0, 0, Z_0) = \{(q_0, xZ_0)\}$

$\delta(q_0, 0, x) = \{(q_0, xx)\}$

$\delta(q_0, 1, x) = \{(q_1, \epsilon)\}$

$\delta(q_1, 1, x) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, x) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}$ construct a CFG.

V – $[q, A, P]$ $\quad q, P \in Q$, $A \in \Gamma$

V – $\{[q_0, x, q_0], [q_0, x, q_1], [q_1, x, q_0], [q_1, x, q_1]$;

$[q_0, Z_0, q_0], [q_0, Z_0, q_1], [q_1, Z_0, q_0], [q_1, Z_0, q_1]\}$

T – $\{0, 1\}$

P ⇒ $S \to [q_0, Z_0, q_0]$ useless ✓

$S \to [q_0, Z_0, q_1]$

$\delta(q_0, 0, Z_0) = \{(q_0, xZ_0)\}$

$[q_0, Z_0, q_0] \to 0[q_0, x, \underline{q_0}][\underline{q_0}, Z_0, q_0]$ X

⇒ $\underline{[q_0, Z_0, q_0]} \to 0[q_0, x, \underline{q_1}][\underline{q_1}, Z_0, q_0]$ x

$[q_0, Z_0, q_1] \to 0[q_0, x, \underline{q_0}][\underline{q_0}, Z_0, q_1]$

$[q_0, Z_0, q_1] \to 0[q_0, x, \underline{q_1}][\underline{q_1}, Z_0, q_1]$

$\delta(q_0, 0, X) = \{(q_0, XX)\}$

$[q_0, X, q_0] \rightarrow 0\ [q_0, X, q_0][q_0, X, q_0]$ ✗

$[q_0, X, q_0] \rightarrow 0\ [q_0, X, q_1]\underline{[q_1, X, q_0]}$ ✗

$\Rightarrow [q_0, X, q_0] \rightarrow 0\ [q_0, X, q_0][q_0, X, q_1]$ ✗

$[q_0, X, q_1] \rightarrow 0\ [q_0, X, q_0][q_0, X, q_1]$

$[q_0, X, q_1] \rightarrow 0\ [q_0, X, q_1][q_1, X, q_1]$

$\delta(q_0, 1, X) = \{(q_1, \epsilon)\}$

$[q_0, X, q_1] \rightarrow 1$

$\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$

$[q_1, X, q_1] \rightarrow 1$

$\delta(q_1, \epsilon, X) = \{(q_1, \epsilon)\}$

$[q_1, X, q_1] \rightarrow \epsilon$

$\delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}$

$[q_1, Z_0, q_1] \rightarrow \epsilon$

There are no productions for the variable $[q_1, X, q_0]$ and $[q_1, Z_0, q_0]$ as all the productions for $[q_0, X, q_0]$ and $[q_0, Z_0, q_0]$ contains the $[q_1, X, q_0]$ / (or) $[q_1, Z_0, q_0]$ on the right of production. There's no terminal string derived from $[q_0, X, q_0]$ (or) $[q_0, Z_0, q_0]$

→ Delete all the productions involving one of these 4 variables on either right or left of production. Hence, we get grammar as

$S \rightarrow [q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \rightarrow 0[q_0, X, q_1][q_1, Z_0, q_1]$

$[q_0, X, q_1] \rightarrow 0[q_0, X, q_1][q_1, X, q_1]$

$[q_0, X, q_1] \rightarrow 1$

$[q_1, X, q_1] \rightarrow 1$

$[q_1, X, q_1] \rightarrow \epsilon$

$[q_1, Z_0, q_1] \rightarrow \epsilon$.

2) construct a CFG G which accepts N(A), where $A = (\{q_0, q_1\}, \{a, b\}, \{Z_0, Z\}, \delta, q_0, Z_0, \phi)$ and δ is given by

$\delta(q_0, b, Z_0) = (q_0, ZZ_0)$ $\quad$ $\delta(q_0, a, Z) = (q_1, Z)$

$\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$ $\quad$ $\delta(q_1, b, Z) = (q_1, \epsilon)$

$\delta(q_0, b, Z) = (q_0, ZZ)$ $\quad$ $\delta(q_1, a, Z_0) = (q_0, Z_0)$

V - $[q, A, P]$ $\quad q, P \in Q$, $A \in \Gamma$

V - $[q_0, Z_0, q_0][q_0, Z_0, q_1][q_1, Z_0, q_0][q_1, Z_0, q_1]$

$[q_0, Z, q_0][q_0, Z, q_1][q_1, Z, q_0][q_1, Z, q_1]$

T - $\{a, b\}$

$S \Rightarrow [q_0, Z_0, q_0]$

$[q_0, Z_0, q_1]$

(i) $\delta(q_0, b, Z_0) = (q_0, ZZ_0)$

$[q_0, Z_0, q_0] \longrightarrow b[q_0, Z, q_0][q_0, Z_0, q_0]$

$[q_0, Z_0, q_0] \longrightarrow b[q_0, Z, q_1][q_1, Z_0, q_0]$

$[q_0, Z_0, q_1] \longrightarrow b[q_0, Z, q_0][q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \longrightarrow b[q_0, Z, q_1][q_1, Z_0, q_1]$

(ii) $\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$

$[q_0, Z_0, q_0] \rightarrow \epsilon$

(iii) $\delta(q_0, b, Z) = (q_0, ZZ)$

$[q_0, Z, q_0] \rightarrow b[q_0, Z, q_0][q_0, Z, q_0]$

$[q_0, Z, q_0] \rightarrow b[q_0, Z, q_1][q_1, Z, q_0]$

$[q_0, Z, q_1] \rightarrow b[q_0, Z, q_0][q_0, Z, q_1]$

$[q_0, Z, q_1] \rightarrow b[q_0, Z, q_1][q_1, Z, q_1]$

(iv) $\delta(q_0, a, Z) = (q_1, Z)$

$[q_0, Z, q_0] \rightarrow a[q_1, Z, q_0]$

$[q_0, Z, q_1] \rightarrow a[q_1, Z, q_1]$

(v) $\delta(q_1, b, Z) = (q_1, \epsilon)$

$[q_1, Z, q_1] \rightarrow \epsilon$

(vi) $\delta(q_1, a, Z_0) = (q_0, Z_0)$

$[q_1, Z_0, q_0] \rightarrow a[q_0, Z_0, q_0]$

$[q_1, Z_0, q_1] \rightarrow a[q_0, Z_0, q_1]$

3) The PDA is given below, construct CFG

$A = (\{q_0, q_1\}, \{0, 1\}, \{S, A\}, \delta, q_0, S, \phi)$

where δ is given by

$\delta(q_0, 1, S) = (q_0, AS)$

$\delta(q_0, \varepsilon, S) = (q_0, \varepsilon)$

$\delta(q_0, 1, A) = (q_0, AA)$

$\delta(q_0, 0, A) = (q_1, A)$

$\delta(q_1, 1, A) = (q_1, \varepsilon)$

$\delta(q_1, 0, S) = (q_0, S)$.

CFG

$V \Rightarrow [q_0, S, q_0], [q_0, S, q_1], [q_1, S, q_1], [q_1, S, q_0]$

$[q_0, A, q_0], [q_0, A, q_1], [q_1, A, q_0], [q_1, A, q_1]$

$T \Rightarrow \{0, 1\}$

$S \rightarrow [q_0, S, q_0]$

$S \rightarrow [q_0, S, q_1]$

(i) $\delta(q_0, 1, S) = (q_0, AS)$

$[q_0, S, q_0] \longrightarrow 1[q_0, A, q_0][q_0, S, q_0]$

$[q_0, S, q_0] \longrightarrow 1[q_0, A, q_1][q_1, S, q_0]$

$[q_0, S, q_1] \longrightarrow 1[q_0, A, q_0][q_0, S, q_1]$

$[q_0, S, q_1] \longrightarrow 1[q_0, A, q_1][q_1, S, q_1]$

(ii) $\delta(q_0, \varepsilon, S) = (q_0, \varepsilon)$

$[q_0, S, q_0] \longrightarrow \varepsilon$

(iii) $\delta[q_0, 1, A] = (q_0, AA)$

$[q_0, A, q_0] \rightarrow 1[q_0, A, q_0][q_0, A, q_0]$

$[q_0, A, q_0] \rightarrow 1[q_0, A, q_1][q_1, A, q_0]$

$[q_0, A, q_1] \rightarrow 1[q_0, A, q_0][q_0, A, q_1]$

$[q_0, A, q_1] \rightarrow 1[q_0, A, q_1][q_1, A, q_1]$

(iv) $\delta(q_0, 0, A) = (q_1, A)$

$[q_0, A, q_0] \rightarrow 0[q_1, A, q_0]$

$[q_0, A, q_1] \rightarrow 0[q_1, A, q_1]$

(v) $\delta(q_1, 1, A) = (q_1, \varepsilon)$

$[q_1, A, q_1] \rightarrow 1$

(vi) $\delta(q_1, 0, S) = (q_0, S)$

$[q_1, S, q_0] \rightarrow 0[q_0, S, q_0]$

$[q_1, S, q_1] \rightarrow 0[q_0, S, q_1]$

a) construct CFG for given PDA.

$M = (\{q_0, q_1\}, \{a, b\}, \{a, Z_0\}, \delta, q_0, Z_0, \phi)$

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, ba)$

$\delta(q_1, b, a) = (q_1, a)$

$\delta(q_1, a, a) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

$V \Rightarrow [q_0, a, q_0], [q_0, a, q_1], [q_1, a, q_0], [q_1, a, q_1]$
$[q_0, b, q_0], [q_0, b, q_1], [q_1, b, q_0], [q_1, b, q_1]$

$T \Rightarrow \{a, b\}$

$S \rightarrow [q_0, Z_0, q_0]$

$S \rightarrow [q_0, Z_0, q_1]$

(i) $\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$[q_0, Z_0, q_0] \rightarrow a[q_0, a, q_0][q_0, Z_0, q_0]$

$[q_0, Z_0, q_0] \rightarrow a[q_0, a, q_1][q_1, Z_0, q_0]$

$[q_0, Z_0, q_1] \rightarrow a[q_0, a, q_0][q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \rightarrow a[q_0, a, q_1][q_1, Z_0, q_1]$

(ii) $\delta(q_0, a, a) = (q_0, aa)$

$[q_0, a, q_0] \rightarrow a[q_0, a, q_0][q_0, a, q_0]$

$[q_0, a, q_0] \rightarrow a[q_0, a, q_1][q_1, a, q_0]$

$[q_0, a, q_1] \rightarrow a[q_0, a, q_0][q_0, a, q_1]$

$[q_0, a, q_1] \rightarrow a[q_0, a, q_1][q_1, a, q_1]$

(iii) $\delta(q_0, b, a) = (q_1, ba)$

$[q_0, a, q_0] \rightarrow b[q_1, b, q_0][q_0, a, q_0]$

$[q_0, a, q_0] \rightarrow b[q_1, b, q_1][q_1, a, q_0]$

$[q_0, a, q_1] \rightarrow b[q_1, b, q_0][q_0, a, q_1]$

$[q_0, a, q_1] \rightarrow b[q_1, b, q_1][q_1, a, q_1]$

(iv) $\delta(q_1, b, a) = (q_1, a)$

$[q_1, a, q_0] \rightarrow b[q_1, a, q_0]$

$[q_1, a, q_1] \rightarrow b[q_1, a, q_1]$

(v) $\delta(q_1, a, a) = (q_1, \varepsilon)$

$[q_1, a, q_1] \rightarrow a$

(vi) $\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)$

$[q_1, z_0, q_1] \rightarrow \varepsilon$

5) $\delta(s, a, x) = (s, Ax)$

$\delta(s, b, A) = (s, AA)$

$\delta(s, a, A) = (s, AA)$

$M = (\{s, s\}, \{a, b\}, \{A, x\}, \delta, s, A/x, \phi)$

(i) $\cancel{\delta(}$ $[s, x, s] \rightarrow a[s, A, s][s, x, s]$

$[s, x, s] \rightarrow a[s, A, s][s, x, s]$

$[s, x, s] \rightarrow a[s, A, s][s, x, s]$

$[s, x, s] \rightarrow a[s, A, s][s, x, s]$

(ii) $\delta(s, b, A) - (s, AA)$

$[s, A, s] \rightarrow b[s, A, s][s, A, s]$

$[s, A, s] \rightarrow b[s, A, s][s, A, s]$

$[s, A, s] \rightarrow b[s, A, s][s, A, s]$

$[s, A, s] \rightarrow b[s, A, s][s, A, s]$

(iii) $\delta(s, a, A) \cdot (s, AA)$

$[s, A, s] \rightarrow a[s, A, s][s, A, s]$

$[s, A, s] \rightarrow a[s, A, s][s, A, s]$

$[s, A, s] \rightarrow a[s, A, s][s, A, s]$

$[s, A, s] \rightarrow a[s, A, s][s, A, s]$

→ DPDA