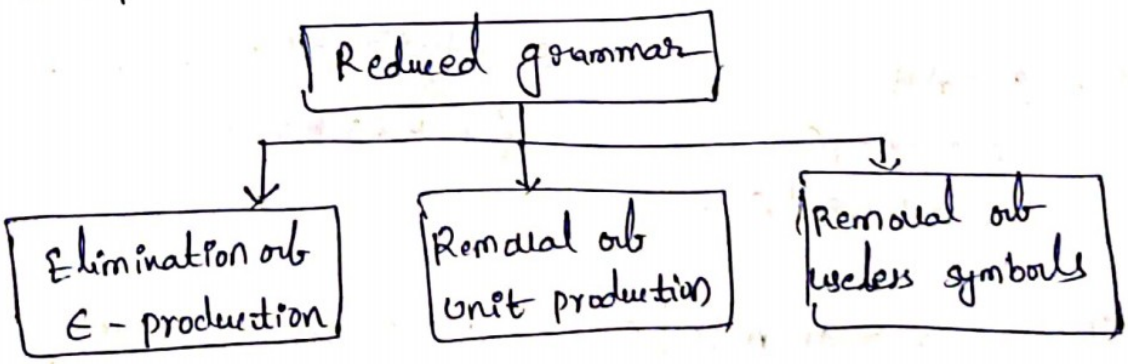
Minimization of Context Free Grammar:→

→ We have seen various languages can effectively be represented by CFG. all the grammars are not always optimized.

→ That means grammar may consist of some extra symbols (non-terminals). Having extra symbols unnecessary increase the length of grammar.

→ The properties of reduced grammar are given

1) each variable (i.e non-terminal) and each terminal of G appears in the derivation of some word in L.
2) There should not be any production as $X \to Y$ where X and Y are non-terminals.
3) if ϵ is not in the language L then there need not be the production $X \to \epsilon$.



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Elimination of ϵ productions from Grammar:-

→ In CFG, if at all there is ϵ production we can remove it, without changing the meaning of the Grammar. Thus ϵ productions are not necessary in a grammar.

Ex:-

* (1) $S \to 0S|1S|\epsilon$.

Sol:- $S \to 0S.$ } $S \to 0$. ∴ $\boxed{S \to 0S|1S|0|1}$ ✓
$S \to \epsilon$ }

$S \to 01$ } $S \to 1$
$S \to \epsilon$ }

* (2) $S \to asb|aAb$
$A \to \epsilon$.

Sol:- $S \to asb|ab|a\cancel{A}b$.

$S \to aAb$ → no production of A ∴ remove it.

∴ $S \to asb|ab$.

* (3). $S \to AB$
$A \to aAA|\epsilon$
$B \to bBB|\epsilon$.

ϵ production are. ⇓ {A, B, S}

$S \to AB$.

Sol:- $S \to AB|A|B|\epsilon$
$A \to aAA|aA|a$.
$B \to bBB|bB|b$.

∴ 'S' is a starting symbol. then only we are going to be added 'ϵ' case. otherwise not.
'ϵ' ϵ L(G) ✓

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④ $S \to AbaC$
$A \to BC$
$B \to b \mid \epsilon$
$C \to D \mid \epsilon$
$D \to d$.

Sol: $\{B, C, A\}$. $A \to B \mid \epsilon$ then $A \to \epsilon$.

$S \to Abac \mid bac \mid Aba \mid ba$.
$A \to Bc \mid B \mid C$
$B \to b$
$c \to D$
$D \to d$.

⑤. Remove the ϵ productions from the following CFG by preserving meaning of it.

$S \to XY*$
$X \to 0X \mid \epsilon$
$Y \to 1Y \mid \epsilon$.

Sol: $\{X, Y, S\}$.

$S \to XY \mid YX \mid XX \mid X \mid Y \mid \epsilon$.
$X \to 0X \mid 0$
$Y \to 1Y \mid 1$

⑥ For the CFG given below remove the ϵ-pro.

$S \to aSa$
$S \to bSb$
$S \to \epsilon$.

Sol: $S \to asa \mid bsb \mid aa \mid bb$

(7) Eliminate the ε-pro. from the CFG

$A \to 0B1 \mid 1B1$

$B \to 0B \mid 1B \mid \epsilon$.

Sol:- $A \to 0B1 \mid 1B1 \mid 01 \mid 11$

$B \to 0B \mid 1B \mid 01$

(8) . $S \to XY$

$X \to Zb$

$Y \to bW$

$Z \to AB$

$W \to Z$

$A \to aA \mid bA \mid \epsilon$

$B \to Ba \mid Bb \mid \epsilon$.

Sol' { A, B, Z, W } → remove it (single production)
(ε) (ε) (ε) (ε)

$S \to XY$

$X \to b$

$Y \to b$.

$A \to aA \mid bA \mid a \mid b$

$B \to Ba \mid Bb \mid a \mid b$

'S' start symbol. $S \to XY$. not reached A, B (no chance to produce)

$S \to XY$

$X \to b$

$Y \to b$.

$\therefore S \to ab$

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(9) Consider the CFG, eliminating ∈ pro.

$S \rightarrow PQR \mid QQ \mid OR \mid RQP$

$P \rightarrow RO \mid IR \mid RR \mid RQP$

$Q \rightarrow QO \mid PQ \mid \in$

$R \rightarrow OP \mid QQQ$.

Sol:

$Q \rightarrow \in$.

$R \rightarrow QQQ \rightarrow \in$.

$P \rightarrow RR \rightarrow \in$.

∴ modify the rules P(or)Q(or)R become ∈.

$S \rightarrow PQR \mid O \mid OR \mid QQ \mid RQP \mid \in$.

$P \rightarrow RO \mid O \mid IR \mid I \mid RR \mid RQP$.

$Q \rightarrow QO \mid O \mid PQ$.

$R \rightarrow OP \mid O \mid QQQ$.

Removing unit productions:—

→ The unit productions are the productions in which one non-terminal gives another non-terminal.

For example:—

$X \rightarrow Y$

$Y \rightarrow Z$

$Z \rightarrow X$

then X, Y, Z are unit productions. To optimize the grammar we need to remove the unit productions.

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→ if $A \overset{*}{\Rightarrow} B$ is a unit production and $B \to X_1X_2X_3 \cdots X_n$ then while removing $A \to B$ Production we should add a rule $A \to X_1X_2X_3 \cdots X_n$.

* (1) If the CFG is as below

$S \to Aa/B$
$B \to A/bb.$
$A \to a/bc/B$. the remove the unit productions.

Sol:

$S \to B \to bb$. and $S \to B \to A \to a, bc$
$B \to A \to a, bc.$
$A \to B \to bb.$

∴ Final productions $S \to Aa/bb/a/bc$
$B \to a/bc/bb$
$A \to a/bc/bb.$

* (2)

$S \to AB$
$A \to a$
$B \to c/b$
$C \to D$
$D \to E$
$E \to a.$

Sol:-

$B \to C \to D \to E \to a.$

∴ $S \to AB$
$A \to a$
$B \to b/a.$
$C \to a$
$D \to a, E \to a.$

∴ C, D, E are not reachable.

∴ $S \to AB$
$A \to a$
$B \to b/a.$

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③ if the CFG is as below

$S \to 0A \mid 1B \mid C$

$A \to 0S \mid 00$

$B \to 1 \mid A$

$C \to 01$ remove the unit productions.

Sol:-

$S \to C \to 01$

$B \to A \to 00, \to 0S$.

∴ $S \to 0A \mid 1B \mid 01$

$A \to 0S \mid 00$

$B \to 1 \mid 0S \mid 00$

$C \to 01$.

④ optimize the CFG given below by reducing the grammar. S is a start symbol.

$S \to A \mid 0C1$

$A \to B \mid 01 \mid 10$

$C \to \epsilon \mid CD$.

Sol: $C \to \epsilon$. so, $S \to A \mid 01$ (ϵ-production)

$S \to A \to B \to$ B is not defined further more.

∴ $S \to 01 \mid 10$

$C \to \epsilon$. is null production.

$C \to CD$, B & D are useless symbols.

Final CFG as

$S \to 01 \mid 10$

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⑤ Eliminate the unit productions from following grammar.

$S \to AB$
$A \to a$
$B \to C \mid b$
$C \to D$
$D \to E \mid bC$
$E \to d \mid Ab$.

Sol: $B \to C \to D \to E \to d \mid Ab$.

replaced value $D \to d \mid Ab \mid bc$.

Similarly $C \to D$

$C \to d \mid Ab \mid bc$.

$\therefore B \to d \mid Ab \mid bc \mid b$.

∴ removed unit productions.

$S \to AB$
$A \to a$
$B \to d \mid Ab \mid bc \mid b$
$C \to d \mid Ab \mid bc$
$D \to d \mid Ab \mid bc$
$E \to d \mid Ab$.

→ There is no path for DAE. So remove useless symbols.

$S \to AB$
$A \to a$
$B \to d \mid Ab \mid bc \mid b$
$C \to d \mid Ab \mid bc$. //

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⑥ simplify the grammar

$= \{ \{S, A, B, C, E\}, \{a, b, c\}, P, S \}$

where P is.

$S \to AB$
$A \to a$
$B \to b$
$B \to C$
$E \to C / \wedge (E)$

Soln Null pro. $\to$ $E \to \in$.

is a useless symbol bcz E cannot be derived from Start symbol.

$B \to C$ is a unit production. There is no rule for derived 'c'. Hence we will eliminate this production.

$\therefore$ so

$S \to AB$
$A \to a$
$B \to b$ //.

⑦. Reduce the grammar G1 given by

$S \to aAa$
$A \to Sb / bCa / DaA$
$C \to abb / DD$
$E \to aC$
$D \to aDA$.

Soln The productions C & E are not reachable.

$D \to aDA \to aaDAA \to aaaDAAA \to \cdots\cdots$

D is not deriving any terminal symbol. i.e useless symbol 'D'.

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$\therefore$ $S \to aAa$
$S \to sb|bcc$. //.

Removal of useless symbols:—

→ Any symbol is useful when it appears on the right hand side, in the production rule and generates some terminal string. if no such derivation exists then it is supposed to be a useless symbol.

→ A symbol p is useful if there exists some derivation in the form.

$$S \overset{*}{\Rightarrow} \alpha p \beta.$$

and $\alpha p \beta \overset{*}{\Rightarrow} w$.

Then p is said to be useful symbol.

* → every variable should part of a string
* → reachable or) not variby.

(1) $S \to AB|a$
$A \to Bc|b$
$B \to aB|C$
$C \to ac|B$. for removing useless symbols.

Sol: First find out useful symbols.

$\{a, b, S, A\}$.

$S \to a$.
$A \to b$.

$B \to aB|C$ → not useful.
→ not useful so B is not useful.

$C \to ac|B$ → not in the set so C is not useful.
→ not in the set.

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$S \to AB \mid a$.
↳ not in the set.

So. $S \to a$.

$A \to BC \mid b$.
not in the set.

So $A \to b$

∴ Final productions. $S \to a$
$A \to b$.

Start symbol S only terminal 'a'. never reached

A.

∴ Final productions $S \to a$.

(2) $S \to AB \mid AC$.
$A \to aAb \mid bAa \mid a$
$B \to bbA \mid aaB \mid AB$
$C \to abcA \mid aDb$
$D \to bD \mid ac$. Find useless symbols.

Sol' Set = {a, b, A, B, S}.

∴ $S \to AB$
$A \to aAb \mid bAa \mid a$.
$B \to bba \mid aaB \mid AB$.

$B \to bbA$ {a, b, A} combination
i.e. reason include 'B' also.
$S \to AB$.
A, B are the set
i.e S is also included
1) $S \to AC$ ✗.
2)

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(3) $S \to ABc \mid BaB$

$A \to aA \mid Bac \mid aaa$

$B \to bBb \mid a$

$C \to CA \mid AC$. remove useless symbols.

Sol:

$\{a, b, A, B, S\}$

$\therefore$ $S \to BaB$

$A \to aA \mid aaa$

$B \to bBb \mid a$.

Start symbol S. $S \to BaB$ never reached A.

$\therefore$ $S \to BaB$

$B \to bBb \mid a$.

(4)

$S \to A11B \mid 11A$

$S \to 1B \mid 11$

$A \to 0$

$B \to BB$

Sol:

$\{0, 1, S, A\}$

$\therefore$ $S \to 11A$

$S \to 11$

$A \to 0$ //

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(5) $S \to AB \mid CA$
$A \to a$
$B \to BC \mid AB$
$C \to aB \mid b$.

Sol: $\{a, b, A, C, S\}$

$\therefore$ $S \to CA$
$A \to a$
$C \to b$. //

(6) $S \to aA \mid bB$
$A \to aA \mid a$
$B \to bB$
$D \to ab \mid Ea$
$E \to ac \mid d$.

Sol: $\{a, b, d, A, E, S, D\}$

$S \to aA$
$A \to aA \mid a$
$D \to ab \mid Ea$
$E \to d$.

start symbol S. $S \to aA$
D & E are not reachable. Then remove it.

$\therefore$ $S \to aA$
$A \to aA \mid a$ //.

(7) $S \to as \mid A \mid C$
$A \to a$
$B \to aa$
$C \to acb$.

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Sol:- {a, b, A, B, S}

∴ S → aS | A
A → a
B → aa

B is never reached.

S → aS | A
A → a. //

(8) S → aA | a | Bb | cC
A → aB
B → a | Aa
C → cCD
D → ddd.

Sol: {a, b, c, d, B, A, S, D}

∴ S → aA | a | Bb
A → aB
B → a | Aa
D → ddd

D is never reached.

S → aA | a | Bb
A → aB
B → a | Aa . //.

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(9) Consider the grammar.

$S \to ABC \mid BaB$

$A \to aA \mid BaC \mid aaa$

$B \to bbb \mid a \mid D$

$C \to CA \mid AC$

$D \to \epsilon$.

Eliminate null, unit and useless productions.

Sol: $D \to \epsilon$

$B \to bbb \mid a$

$\{a, b, B, A, S\} \to$ useful symbols.

$\therefore S \to BaB$

$A \to aA \mid aaa$

$B \to bbb \mid a$.

S is start symbol. never reached A.

$\therefore S \to BaB$

$B \to bbb \mid a$ //

(10) $S \to AaA \mid CA \mid BaB$

$A \to aaBa \mid CDA \mid aa \mid DC$

$B \to bB \mid aAB \mid bb \mid as$

$C \to Ca \mid bC \mid D$

$D \to bD \mid \epsilon$

Sol: $D \to \epsilon$

so. $C \to Ca \mid bC$

There is no unit productions.

$\{a, b, B, A, S\}$

$\therefore S \to AaA \mid BaB$

$A \to aaBa \mid aa$

$B \to bB \mid aAB \mid bb \mid as$ //

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(11) $S \to ASB \mid \epsilon$
$A \to aAS \mid a$
$B \to SbS \mid A \mid bb$.

Sol: $\{a, b, AB, S\}$

$\because S \to \epsilon$. $\therefore$ $S \to AB \mid ASB$
$A \to aA \mid a \mid aAS$
$B \to b \mid A \mid bb \mid Sb \mid bS \mid SbS$

(12) $S \to aA \mid aB$
$A \to aaA \mid B \mid \epsilon$
$B \to b \mid bB$
$D \to B$.

Sol: $A \to \epsilon$.

$\{a, b, B, S, A\}$

$S \to aA \mid aB$
$A \to aaA \mid B$
$B \to b \mid bB$ //.

$A \to \epsilon$ means,
$S \to aA \mid aB \mid a$.
$A \to aa \mid aaA \mid B$
$B \to b \mid bB$

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