UNIT-1

RANDOM VARIABLES

→ The theory of probability is one of the most useful and intresting branches of modern mathematics by its applications in many fields of learning such as insurance, statistics, biological sciences, physical sciences, engineering etc.

→ If any experiment is repeated under similar and homo-geneous conditions we generally come across two types of situations.

1) The net result which is generally known as outcome is unique or certain.
2) The net result is not unique but may be one of the several possible outcomes.
3) The situations covered by one are known as determ-inistic (or) predictable and situations covered by ② are known as probabilistic (or) unpredictable

Random Experiment :-

→ If an Experiment is conducted any no. of times under essentially identical conditions. There is a set of all possible outcomes associated with it. if the result is not certain and is any one of the several possible outcomes. The Experiment is called as Random Experiment or Random trial.

→ The outcomes are known as elementary events and set of outcomes is an event.

* **Exhaustive Events :**

→ All possible events in any trials are known as Exhaustive Events. Eg: throwing a dice n times there are 6^n Exhaustive Events

* **Mutually Exclusive Event :**

→ If two or more of them cannot happen simultan-neously in a trial then the event is called Mutually Exclusive Event.

Eg: 1) In tossing a coin the events heads, tails are mutually exclusive.

* Evually likely Events:
→ Events are said to be evually likely when there is no reason to expect any of them rather than any one of the other.
Eg: Throwing a die, chance of getting 1, 2, ... 6 are Evually Likely.

* Independent Events:
→ Several events are said to be independent if the happening or the non-happening is not effected by concern of the occurance of any one of the remaining Events.
1) An event that always happens is called Certain Event.
2) An Event that never happens is called an impossible Event.

⇒ Probability (mathematical Expression)
→ In a Random Experiment let there be n mutually Exclusive and evually likely elementary Events let 'E' be an Event in the Experiment. If 'm' Elementary events for E (favourable) then the probability of 'E' is defined as.

$$P(E) = \frac{\text{No. of elementary events in } E}{\text{Total No. of elementary events in Random Exp.}}$$

$$\therefore \boxed{P(E) = m/n}$$

Note:
1) If E denotes probability of an event and $\bar{E}$ denotes the event of non-occurance of E then, $\boxed{P(E) + P(\bar{E}) = 1}$

2) If $P(E) = 1$, the event 'E' is Said to be Certain Event and If $P(E) = 0$ the Event is said to be impossible Event.

Problems:

① Five digit numbers are formed with 0, 1, 2, 3, 4 Find P(E) of getting 2 in ten's place and 0 in unit's place always (no repetition).

Sol: Total Five digit numbers using 0, 1, 2, 3, 4. is

$n = (5-1)\times 4\times 3\times 2\times 1 = 96$

$\boxed{n=96} \rightarrow$ Total outcomes

let E be the Event of getting a number with 0 in unit's place and 2 in one's.

$(\underset{3\,ways}{\uparrow}\ \underset{2\,ways}{\underline{\ \downarrow\ }}\ \underset{1\,way}{\uparrow}\ \underline{2}\ \underline{0}) \Rightarrow 3\times 2\times 1\times 1 = 6$

$\boxed{\therefore m=6}$

$\therefore P(E) = m/n = 6/96 = 1/16$ //

② What is the probability of leap year to have 52 mondays and 53 Sundays.

Sol: → leap year Consists of 366 days ⇒ (52 weeks + 2 day)

→ The possible days may be

⇒ Tue & wed, mon & tue, thu & fri, fri & sat, sat & sun, sun & mon wed & thu.

Total Favourable cases = 7 = n

let E be is the Event of having 52 mondays and 53 Sundays

⇒ Total No. of Favourable cases

$\boxed{m=1}$

$P(E) = m/n = 1/7$.

③ Determine the probability in each of the cases

(i) A non defective bolt will be found out of 600 bolts. If already examined the 12 were defective

$P(D) = \frac{12}{600} = \frac{1}{50}$; $P(\bar{D}) = 1 - P(D)$

Probability of defective bolt $P(D) = 1/50$

⇒ probability of non defective bolt.

$P(\bar{D}) = 1 - P(D) = 1 - 1/50 = 49/50$

* Simple Event:

→ An event in a trial that can't be further split is called a simple event or an elementary event.

* Sample Space:

→ Set of all possible simple events in a trial is called a sample space. Each element of this set is called a Sample Count. The sample space is denoted by 'S'.

Note:

generally $S = \{e_1, e_2, e_3 \ldots e_n\}$ are known as sample points of the experiment.

⇒ Notations:

Event	Meaning in set theory
1) Impossible Event	ϕ
2) Certain Event	S
3) atleast one of events A/B occurs	$A \cup B$
4) A occurs B doesn't	$A \cap B'$
5) B occurs A doesn't	$B \cap A'$
6) Both A & B occurs	$A \cap B$
7) Neighter A occur nor B occurs	$A' \cap B'$

⇒ probability Axiomatic approach

→ let 'S' be a finite sample space a real valued function 'f' from the power set of 'S' x power set 'R' is called probability fun^n on 'S'. It following 3 axioms satisfies.

1) Axiom of positivity i.e., $P(E) \geq 0$ for every Subset $E(S)$.

2) Axiom of certainity i.e., $P(S) = 1$.

3) Axiom of union If E_1, E_2 are disjoint subsets of S, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$ $\{ \{A_i\}_{i=1}^{n}$ is a sequence of mutually Exclusive Events $\}$

Problems:

① A class consists of 6 girls and 10 boys if a comittee of 3 is chosen at random from class. Find:

(i) probability of 3 boys are selected.

(ii) probability of 2 girls are selected.

Sol: (i) Total no. of outcomes $= {}^{16}C_3 = n$

Favourable outcomes $= {}^{10}C_3 = m$

$P(E) = m/n = {}^{10}C_3 / {}^{16}C_3 = 3/14$

(ii) $P(E) = ({}^{10}C_1 \times {}^{6}C_2) / {}^{16}C_3 = 15/56$

Properties:

(i) $P(\phi) = 0$

(ii) If A & B be any sets (events)

1) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

2) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

→ **Addition theorem of probability:**

→ If A & B are any two events in S (need not be disjoint) then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

1) If E_1 & E_2 are two mutually exclusive (disjoint sets) events then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$P(\overline{E_1 \cup E_2}) = P(\bar{E_1}) - P(\bar{E_2})$.

→ **Conditional event:**

→ If E_1, E_2 are any two events of sample space S and If E_2 occurs after occurance of E_1 then event of occurance of E_2 after event E_1 is called conditional event of E_2 for

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given event. it is denoted by E_2/E_1.

→ Conditional probability:

If E_1, E_2 are any two events in sample space S and $P(E_1) \neq 0$, $P(E_2) \neq 0$. Then probability of E_2 after the event E_1 has occured is called Conditional probability of Event E_2 for given E_1

Denoted by $P(E_2/E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{n(E_2 \cap E_1)}{n(E_1)}$.

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{n(E_1 \cap E_2)}{n(E_2)}.$$

→ Multiplication theorem:

→ In a Random exp E_1, E_2 are two events with such that $P(E_1) \neq 0$, $P(E_2) \neq 0$ then

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$$

$$P(E_2 \cap E_1) = P(E_2) \cdot P(E_1/E_2).$$

→ Independent event:

→ An Event B is said to be independent of A (A & B are said possible Events). If conditional probability of B/A is $P(B)$ then

$$P(A \cap B) = P(A) \cdot P(B).$$

⇒ If $P(A/B) = P(A)$.

$$P(B \cap A) = P(B) \cdot P(A).$$

→ If B is independent of A then A is also independent of B.

→ The necessary and sufficient condition for two possible events A and B to be independent is:

$$P(A \cap B) = P(A) \cdot P(B).$$

→ Generalized Multiplication law of probability:

→ If $A_1, A_2, \ldots, A_n$ are events in sample space S then ⇒ $P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n)$

$$\Rightarrow P(A_1) \cdot P(A_2/A_1) \cdot P(A_3) \cdot P(A_4/A_3) \ldots$$
$$P(A_n / A_1 \cap A_2 \ldots \cap A_{n-1})$$

→ **Pair wise independent:**

→ A set of events $A_1, A_2 \dots A_n$ are said to be pair wise independent events if probability

$P(A_i \cap A_j) = P(A_i) \cdot P(A_j)$ for $i \neq j$.

where $i = 1, 2 \dots n$

$j = 1, 2 \dots n$.

→ **Mutually independence:**

→ A set of events $A_1, A_2 \dots A_n$ are said to be mutually independent events if.

$P(A_i \cap A_j \cap A_k) = P(A_i) \cdot P(A_j) \cdot P(A_k)$ ∀ $i \neq j \neq k$.

⋮

$P(A_1 \cap A_2 \cap \dots A_n) = P(A_1) \cdot P(A_2) \dots P(A_n)$.

Note:-

1) Total no. of conditions for pair wise independent sets ⇒ $^nC_2$.

2) Total no. of conditions for mutually independent events ⇒ $2^n - 1 - n$.

3) Mutually independent event implies pair wise independent event but converse is not true.

4) If 3 events A, B, C are mutually independent then $A \cup B$ is independent of C

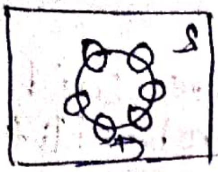
$P((A \cup B) \cap C)) = P(A \cup B) \cdot P(C)$.

→ **Baye's Theorem:**

→ If $E_1, E_2 \dots E_n$ are mutually exclusive and exhaustive events such that probability of $P(E_i) > 0$ for $i = 1, 2 \dots n$ in a sample space S and A is any other event in S intersecting with every E_i such that $P(A) > 0$

→ If E_i is any of the events of $E_1, E_2 \dots E_n$ where $P(E_1), P(E_2), \dots P(E_n)$ and $P(A/E_1) \cdot P(A/E_2) \dots P(A/E_n)$ are known then

$$P(E_k/A) = \frac{P(E_k) \cdot P(A/E_k)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + \dots + P(E_n) P(A/E_n)}$$

⇒  pictorial Representation

problems:

① A Bag 'A' contains 2 white balls, 3 Red balls and Bag 'B' contains 4 white balls, 5 Red balls. One ball is drawn out Random from one of the bag it is found to be Red find the probability that Red ball is drawn from Bag B.

Sol: $$P(B/R) = \frac{P(B)\cdot P(R/B)}{P(A)\cdot P(R/A) + P(B)\cdot P(R/B)} \longrightarrow ①$$

$P(A) = P(B) = 1/2$

$P(R/B) = 5/4$; $P(R/A) = 3/5$

$$① \Rightarrow \frac{1/2 \cdot 5/4}{1/2 \cdot 3/5 + 1/2 \cdot 5/9} = 25/32.$$

② The chance that doctor A will diagnosis disease x correctly 60%. The chance that a patient will die by his treatment after correct diagnosis 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A who had disease x died. What is the chance that disease was diagnosis correctly

Sol: Let 'D_1' be the event that disease 'x' is diagnised correctly by doctor A.

let 'D_2' be the event that a patient of doctor A who had disease x died.

$P(D_1) = 60/100 = 0.6$

$P(\bar{D}_1) = 1 - 0.6 = 0.4$ (wrong diagnosis)

$P(D_2/\bar{D}_1) = 70/100 = 0.7$

$P(D_2/D_1) = 40/100 = 0.4$

By Baye's Theorem:

$$P(D_1/D_2) = \frac{P(D_1)\cdot P(D_2/D_1)}{P(D_1)\cdot P(D_2/D_1) + P(\bar{D}_1)\cdot P(D_2/\bar{D}_1)} = \frac{(0.6)(0.4)}{(0.6)(0.4)+(0.4)(0.7)} = \boxed{6/13}$$

(3) In a certain college 25% of boys and 10% of girls are studying mathematics the girls constitute 60% of the student Body.

i) what is the probability that mathematics is being studied?

ii) If a student is selected random and found to be studied mathematics. Find the probability that student is girl?

(iii) If a student is selected random and found to be studied mathematics find the probability that student is boy?

Sol:

i sol: $P(M) = P(G) \cdot P(M/G) + P(B) \cdot P(M/B)$

$= (0.6)(0.1) + (0.4)(0.25)$

$= 4/25$

$P(G) = \frac{60}{100} = 0.6$

$P(B) = \frac{40}{100} = 0.4$

$P(M/B) = \frac{25}{100} = 0.25$

$P(M/G) = \frac{10}{100} = 0.1$

(ii) $P(G/M) = \frac{P(G) \cdot P(M/G)}{P(G) \cdot P(M/G) + P(B) \cdot P(M/B)}$

$= \frac{(0.6)(0.1)}{(0.6)(0.1) + (0.4)(0.25)}$

$= 3/8$

(iii) $P(B/M) = \frac{P(B) \cdot P(M/B)}{P(B) \cdot P(M/B) + P(G) \cdot P(M/G)}$

$= \frac{(0.4)(0.25)}{(0.4)(0.25) + (0.6)(0.1)}$

$= 5/8$

Random variable:

→ A Real valued function defined on a sample Space 'S' is called R.V.

→ R.V are usually denoted by capital letters of Eng letters and perticular values which random values will take are denoted by corresponding small letters

Eg: Tossing two coins.

$S = \{ HT, TH, TT, HH \}$

$S = \{ s_1, s_2, s_3, s_4 \}$

$s_1 = \{HT\}$
$s_2 = \{TH\}$
$s_3 = \{TT\}$
$s_4 = \{HH\}$

let $X : S \to \mathbb{R}$ defined by X = No. of heads

$X(s_1) = 1$
$X(s_2) = 1$
$X(s_3) = 0$
$X(s_4) = 2$

Range of $X = \{0, 1, 2\}$

→ Random variables are two types:

(i) Descrete R.V.
(ii) Continuous R.V.

* **Descrete R.V:**

→ A real valid funⁿ defined on a descrete sample is called D.R.V.

Eg:- No. of deffectives in a sample of bulbs

* **Continuous R.V.**

→ A R.V is said to be continuous if it takes every values b/w two numbers (or) certain limits.

Eg: height, temperature, weight

→ probability distribution Fun^n :

let X be a R.V, then the probability fun^n associated with X is defined as probability that the outcome of an experiment will be one of the outcomes for which $X(s) \leq x$, $\forall x \in R$ i.e., the function $F_X(x)$ [(or) $F(x)$] defined by

$$F_X(x) = F(x) = P(X \leq x)$$
$$= P\{s : X(s) \leq x\}, \ -\infty < x < \infty$$

is called distribution fun^n of X.

* properties:

① If F is a distribution function of R.V X and if $a < b$ then

* $P(a < X \leq b) = F(b) - F(a)$
* $P(a \leq X \leq b) = P(X = a) + \{F(b) - F(a)\}$
* $P(a < X < b) = [F(b) - F(a)] - P(X = b)$
* $P(a \leq X < b) = [F(b) - F(a)] - P(X = b) + P(X = a)$

② All distribution fun^n are monotonically increasing and lies b/w 0 and 1.

If F is the distribution Fun^n of R.V 'X' then,

* $0 \leq F(x) \leq 1$
* $F(x) < F(y)$ where $x < y$.
* $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$
* $F(\infty) = \lim_{x \to \infty} F(x) = 1$.

Note:

If $P(X=a) = P(X=b) = 0$, then

$\Rightarrow P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$

$= P(a \leq X \leq b) = F(b) - F(a)$

* **Descrete probability distribution:**

(or) **probability mass Function**

→ Suppose, 'X' is a descrete R.V. with possible outcomes $x_1, x_2, \ldots$ the probability of each possible outcome x_i is $P_i = P(X = x_i) = P(x_i)$

→ If the numbers $P(x_i)$ satisfying folloing two conditions:

1) $P(x_i) > 0 \quad \forall \; i \in \mathbb{R}$

2) $\Sigma P(x_i) = 1$

then the function $P(x_i)$ is called probability mass function to x. and the set $\{P(x_i)\}$ $i = 1, 2, \ldots$ is called descreted probability distribution.

* **Descrete distribution Function**

(or) **Cumilative distribution Fun^n**

Suppose 'X' is a descrete R.V then the function $F(x)$ is defined by

$F(x) = P(X \leq x) = \Sigma P(x_i)$

* **probability density Function:**

→ The P.D.F $f_X(x)$ is defined as the derivative of the probability distribution function $f_X(x) = d/dx\,(F_X(x))$

→ Expectation, mean, variance and standard deviation of a probability distribution Function:

* **Expectation :**

The mathematical Expectation or mean or Expected value of 'x' is denoted by $E(x)$ and defined as $E(x) = \sum_{i=1}^{n} p_i x_i$

→ If $g(x)$ is any funⁿ of R.V 'x'.

$$E(g(x)) = \sum_{i=1}^{n} p_i g(x_i)$$

Note : 1) If x is a R.V and 'k' is a constant then ~~PK~~

$$E(x+k) = E(x) + k$$

2) If x, and y any two R.V then

$$E(x+y) = E(x) = E(y)$$

3) If x is R.V and a and b are Constants then

$$E(ax \pm b) = a.E(x) \pm b$$

4) If x and y are two independent R.V, then

$$E(xy) = E(x).E(y)$$

5) $E(1/x)$ and $1/E(x)$ are not same

* **Mean :**

The mean value μ of a discrete distribution funⁿ is

$$\mu = \sum_{i=1}^{n} p_i x_i = E(x).$$

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* **Variance (σ^2):**

It characterizes the variability in the distributions since two distributions with same mean can still have different dispersions of data about their means

i.e., $\sigma^2 = E(x^2) - \mu^2$

* **Standard deviation (σ):**

It is the positive square root of variance.

i.e., $S.D = \sqrt{\sigma^2} = \sigma = \sqrt{E(x^2) - \mu^2}$.

Q: let x denotes the no. of heads in a single toss of four coins, determine

(i) $P(x<2)$,

(ii) $P(1<x\leq 3)$.

(i) sol: Total outcomes $= 2^4 = 16$

$\therefore S = \{$ HHHH, HHHT, HHTT, HTTT, TTTT, HTHT, THTH, THHH, TTHH, TTTH, HHTH, TTHT, THTT, THHT, HTTH, HTHH $\}$

X: No. of heads

Distribution table:

X	0	1	2	3	4
X(S)	1/16	4/16	6/16	4/16	1/16

(i) $P(X<2) = P(X=0) + P(X=1)$

$= 1/16 + 4/16 = 5/16$

(ii) $P(1 < X \le 3) = P(X=2) + P(X=3)$

$= 6/16 + 4/16 = 10/16 = 5/8$

Q: two dies are thrown, let X assigned to each point (a,b) in X, the maximum of its numbers. $X(a,b) = \max(a,b)$

Find probability distribution, X is R.V with $X(S) = \{1,2,3,4,5,6\}$ also find mean and varience of distribution.

Sol: Total outcomes $= 6^2 = 36$

Given condition $X(S) = X(a,b) = \max(a,b)$

⇒ The Number '1' : X(1)

the favourable cases with '1' as maximum is 1

$P(X=1) = 1/36 \longrightarrow \{(1,1), (1,2), (1,3) \ldots (1,6)\}$

⇒ The Number '2' : X(2)

the favourable cases with 2 as max.

$P(X=2) = 3/36$ $\{(1,2), (2,1), (2,2), (2,3) \ldots$

⇒ The number '3' : X(3)

the favourable outcomes '3' as max

$P(X=3) = 5/36$ $\{(1,3), (2,3), (3,3), (3,1), (3,2)\}$

⇒ the number '4' : X(4)

the Favourable out comes '4' as max

$P(X=4) = 7/36$

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$\rightarrow P(x=5) = 9/36$

$\rightarrow P(x=6) = 11/36$

Distribution Table :

X	1	2	3	4	5	6
X(i)	1/36	3/36	5/36	7/36	9/36	11/36

$\rightarrow$ Mean $= \mu = \Sigma x_i p_i$

$$= 1/36 + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36}$$

$$= \frac{161}{36}$$

$\rightarrow$ variance $= \sigma^2 = E(x^2) - \mu^2$

$$= \Sigma P_i x_i^2 - \mu^2$$

$$= \frac{1}{36} + \frac{12}{36} + \frac{45}{36} + \frac{112}{36} + \frac{225}{36} + \frac{396}{36} - \left(\frac{161}{36}\right)^2$$

$$= \frac{791}{36} - \left(\frac{161}{36}\right)^2$$

$$= 21.97 - 20$$

$$= 1.97$$

Q: The R.V 'x' has the probability fun^n

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Sol: (i) determine 'k'

(ii) Evaluate $P(X<6)$, $P(X \geq 6)$, $P(0<X<5)$

(iii) Determine the distribution func^n of X
(iv) Mean (μ)
(v) Variance (σ^2)

(i) $\because \sum_{i=1}^{7} P(x_i) = 1$

$\therefore 9k + 10k^2 = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$10k^2 + 10k - k - 1 = 0$

$10k(k + 1) - (k+1) = 0$

$k = -1 \;;\; k = 1/10$

$$\boxed{\therefore \; k = 1/10}$$

Now, probability distribution table

x	0	1	2	3	4	5	6	7
$P(x)$	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/10^2$	$2/10^2$	$7/10^2 + 1/10$

(ii) $P(X<6)$

$\Rightarrow P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$\Rightarrow 0 + 1/10 + 2/10 + 2/10 + 3/10 + 1/10^2$

$\Rightarrow 8/10 + 1/10^2 = \dfrac{80+1}{100} = \dfrac{81}{100}$

$\Rightarrow P(X \geq 6) = 1 - P(X<6) = 1 - 81/100$

$\Rightarrow$ ~~or~~ $\cancel{P(X=6) + P(X=7)} = 19/100$

$P(X \geq 6) = 19/100$.

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$P(0<x<5)$

$\Rightarrow P(x=1)+P(x=2)+P(x=3)+P(x=4)$

$\Rightarrow 1/10+2/10+2/10+3/10$

$\Rightarrow 8/10$

(iv) mean (μ)

$\mu = \Sigma x_i p_i$

$= 1/10+4/10+6/10+12/10+5/10^2+12/10^2+7\left(\frac{7}{10^2}+\frac{1}{10}\right)$

$\Rightarrow \frac{30}{10}+\frac{66}{10^2} \Rightarrow \frac{366}{100} = 3.66$

(v) variance (σ^2)

$\Rightarrow \Sigma x_i^2 p_i - \mu^2$

$\Rightarrow (1/10)+(2/10)4+(2/10)9+16\times3/10+25/10^2+1\cdot 7^2/10^2$
$+49\left(7/10^2+1/10\right)-\left(\frac{366}{100}\right)^2$

$\Rightarrow \frac{75}{10}+\frac{489}{100}-\left(\frac{366}{100}\right)^2$

$\Rightarrow \frac{750+489}{100}-\left(\frac{366}{100}\right)^2$

$\Rightarrow \frac{1239}{100}-\left(\frac{366}{100}\right)^2$

$\Rightarrow 12.39-13.3956$

$= -1.0056$

Q: A R.V 'x' has the probability distribution

x	0	1	2	3	4	5	6	7	8
$P(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) determine the value 'a'.

(ii) Find $P(X<3)$, $P(X\geq 3)$

(iii) Find distribution fun $F(x)$.

sol > $\sum p(x_i) = 1$

$81a = 1$

$a = 1/81$

→ distribution of x

x	0	1	2	3	4	5	6	7	8
$p(x)$	$1/81$	$3/81$	$5/81$	$7/81$	$9/81$	$11/81$	$13/81$	$15/81$	$17/81$

(ii) $P(X<3) \Rightarrow P(X=0) + P(X=1) + P(X=2)$

$\Rightarrow 1/81 + 3/81 + 5/81$

$\Rightarrow 9/81 = \underline{1/9}$

$P(X\geq 3) = 1 - P(X<3)$

$= 1 - 1/9 = \underline{8/9}$

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Q: A player tosses three fair coins. He wins ₹500 if 3 heads appear, ₹300 if two heads appear, ₹100 if 1 head occurs. On the other hand he looses ₹1500 if three tails occur. Find the Expected game of the player.

Sol: Player tosses 3 fair coins

Let x denotes the game

Range of 'x' is $\{-1500, 100, 300, 500\}$

$S = \{HHH, HHT, HTT, HTH, TTH, THT, TTT, THH\}$

$\Rightarrow$ Probability of all heads $= 1/2^3 = 1/8$

$\Rightarrow$ " " 1 head $= 3/2^3 = 3/8$

$\Rightarrow$ " " 2 heads $= 3/8$

$\therefore P(x=3) = 1/8$

$P(x=2) = 3/8$

$P(x=1) = 3/8$

$P(x=0) = 1/8$

→ distribution of x

x	-1500	100	300	500
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

→ Expected Game of player $= E(x)$

$= \sum p_i x_i$

$\Rightarrow \frac{-1500}{8} + \frac{300}{8} + \frac{900}{8} + \frac{500}{8}$

$\Rightarrow \frac{200}{8} = 25/-$

Q: A player tosses two Fair Coins he wins ₹ 100 If head Appears, ₹ 200 If two head appears on the other hand he loses ₹ 500 If no head appears, determine the Expected value E of the game and is the game Favourable to player

Sol: player tosses 2 Fair coins
let x denotes the game
Range of X is $= \{100, 200, -500\}$

$S = \{HH, HT, TH, TT\}$

→ probability of one head appears $= 2/2^2$
$= 1/2$

→ probability of two " " $= 1/4$

→ probability of no head $= 1/4$

$P(x=1) = 1/2$

$P(x=2) = 1/4$

$P(x=0) = 1/4$

→ distribution of 'x'.

x	-500	100	200
$P(x)$	$1/4$	$1/2$	$1/4$

→ Expected Game of player $= E(x)$

$= \sum P_i x_i$

$$\Rightarrow \frac{-500}{4} + \frac{200}{4} + \frac{200}{4}$$

$\Rightarrow -25$ ₹

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(ii) Continuous Random Variables:

→ **Probability density Function:**

* let $F(x)$ be any continuous function of x so that $F(x)dx$ represents the probability that the variable x lies in interval $(x - dx/2, x + dx/2)$ symbolically it can written as probability of

$\Rightarrow P(x - dx/2 \le X \le x + dx/2)$. then $F(x)$ is called the probability density Function (or) Density Function of variant x

→ The continuous $y = f(x)$ is known as the probability density Curve

Properties:

① $f(x) \ge 0 \quad \forall x \in \mathbb{R}$

② $$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

③ $P(E)$ is given by

$$P(E) = \int_E f(x) \cdot dx$$ is well defined for any event 'E'.

④

→ **Cumilative Distribution Function:**

For a R.V 'X' the C.D.F is denoted by $F(x)$ and defined as $F(x) = P(X \le x)$

$$\Rightarrow F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\, dx,$$

Properties:

① $0 \le F(x) \le 1$

② $F'(x) = f(x) \ge 0$

③ $F(-\infty) = 0$

④ $F(\infty) = 1$

⑤ $F(x)$ is continuous Function of x on the right

⑥ The discontinuties of

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F(x) are Countable.

⑦ $P(a \le x \le b) = \int_a^b f(x)\,dx$

→ **Measures of Central tendency:**

① **Mean (or) Expectation (μ):**

let 'x' be a R.V. then $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$.

In general the mean or expectation of any Function $\phi(x)$ is given by

$$\mu = \int_{-\infty}^{\infty} \phi(x) \cdot f(x)\,dx.$$

② **Median:**

→ It is a point which divides distribution into two Equal parts. If 'x' is defined from a to b, and 'M' is median then $\int_a^M f(x)\,dx = \int_M^b f(x)\,dx = 1/2$, solving it we can obtain Median.

③. **Mode:**

→ It is a value of 'x' for which $f(x)$ is maximum.

⇒ $f'(x) = 0$ & $f''(x) < 0$ where $a < x < b$

④ **Variance (σ^2):**

→ $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$.

⑤ **Mean deviation:**

→ mean deviation about the mean 'μ' is given by $\int_{-\infty}^{\infty}$ ~~text~~ ⇒ $\int_{-\infty}^{\infty} |x-\mu| f(x)\,dx$.

Q: If a R.V has the P.D.F $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x \leq 0 \end{cases}$ find the probabilities that it will take on a value.

(i) between 1 and 3

(ii) > 0.5

i sol: $P(1 \leq x \leq 3) = \int_1^3 f(x)dx = \int_1^3 2e^{-2x}dx$

$= 2 \cdot \left(\frac{e^{-2x}}{-2}\right)_1^3$

$= -(e^{-6} - e^{-2})$

$\Rightarrow \underline{e^{-2} - e^{-6}}$

(ii) sol: $P(x > 0.5) = \int_{0.5}^{\infty} f(x)dx = \int_{0.5}^{\infty} 2e^{-2x}dx$

$= 2\left(\frac{e^{-2x}}{-2}\right)_{0.5}^{\infty}$

$= -(0 - e^{-1})$

$= 1/e$

Q: The probability density $f(x)$ of continuous R.V is given by $f(x) = C.e^{-|x|}$, $-\infty < x < \infty$ show that $C = 1/2$ and find mean and variance of the distribution also find the probability that variant lies b/w blue 0 and 4

$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} c.e^{-|x|}dx = 1$

$\Rightarrow c. \int_{-\infty}^{\infty} e^{-|x|}dx = 1$

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$$\int_{-\infty}^{0} e^{f(x)}\,dx + \int_{0}^{\infty} e^{-x}\,dx = 1/c$$

$$\left(e^{x}\right)_{-\infty}^{0} + \left(\frac{e^{-x}}{-1}\right)_{0}^{\infty} = 1/c \qquad -e^{-x}$$

$$1 - 0 + 0 + 1 = 1/c.$$

$$\boxed{c = 1/2} \qquad u\int v\,dx - \ldots$$

$$\Rightarrow \mu = \int_{-\infty}^{\infty} x\,f(x)\,dx = \int_{-\infty}^{\infty} x\cdot c e^{-|x|}\,dx$$

$$\Rightarrow c\underbrace{\int_{-\infty}^{\infty} x\cdot e^{-|x|}\cdot dx}_{\text{odd Func}} = 0 \qquad \boxed{\mu = 0}$$

$$\Rightarrow \sigma^2 = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - \mu^2 \qquad \int x\cdot x$$

$$\int uv \qquad u\int v - \int u' \int v.$$

$$= c\int_{-\infty}^{\infty} x^2\cdot e^{-|x|}\,dx - 0 \qquad x\cdot\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}$$

$$\frac{x^3}{3} - \frac{x^3}{6}$$

$$= 2c\int_{0}^{\infty} x^2\cdot e^{-|x|}\,dx \qquad \frac{2x^3 - x^3}{6} = x^3/6$$

$$= 2c\left\{\left(e^{-x}\cdot\left(\frac{x^3}{3}\right)\right)_0^{\infty} - \int_{0}^{\infty}\left(-e^{-x}\right)\cdot\frac{x^3}{3}\,dx\right\}$$

$$= \frac{2c}{3}\left\{\left(x^3\cdot e^{-x}\right)_0^{\infty} + \int_0\left(x^3 e^{-x}\right)\right\}$$

$$= 2\cdot 1/2 \int_{0}^{\infty} x^2\cdot e^{-x}\,dx$$

Gamma Function

$$\Gamma n = \int_0^{\infty} x^{n-1}\cdot e^{-x}\,dx$$

$$\Gamma n = (n-1)!$$

$$= \Gamma 3 = (3-1)! = 2$$

$$\therefore \boxed{\sigma^2 = 2} \Rightarrow \boxed{\sigma = \sqrt{2}}$$

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$\Rightarrow P(0 \le x \le 4) = \int_0^4 f(x)\,dx$

$= \int_0^4 c e^{-|x|}\,dx = \frac{1}{2} \int_0^4 e^{-x}\,dx$

$= \frac{1}{2} \left(-e^{-x}\right)_0^4$

$= -\frac{1}{2}\left(e^{-4} - 1\right)$

$= \frac{1-e^{-4}}{2}$

Q: For a Continuous Random variable x whose p.d.f. $f(x) = \begin{cases} cx(2-x) & ; 0 \le x \le 2 \\ 0 & ; \text{otherwise} \end{cases}$

where c is a constant, find 'c', mean and varience of 'x' also find $P(1 \le x \le 2)$

Sol: $\because \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_0^2 f(x)\,dx + \int_2^{\infty} f(x)\,dx$

$\Rightarrow 0 + \int_0^2 cx(2-x)\,dx + 0$

$\Rightarrow c \int_0^2 (2x - x^2)\,dx$

$= c \left\{ \cancel{2}\left(\frac{x^2}{\cancel{2}}\right)_0^2 - \left(\frac{x^3}{3}\right)_0^2 \right\}$

$= c \left\{ 4 - \frac{8}{3} \right\}$

$= \frac{4c}{3} = 1$

$\boxed{c = 3/4}$

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$\Rightarrow$ Mean $(\mu) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{2} x f(x)\,dx + \int_{2}^{\infty} f(x)\,dx.$$

$$\Rightarrow 0 + \int_{0}^{2} x \cdot \frac{3}{4} \cdot x(2-x)\,dx + 0$$

$$\Rightarrow \frac{3}{4} \int_{0}^{2} x^2(2-x)\,dx$$

$$\Rightarrow \frac{3}{4} \int_{0}^{2} (2x^2 - x^3)\,dx$$

$$\Rightarrow \frac{3}{4} \left(2\frac{x^3}{3} - \frac{x^4}{4}\right)_0^2$$

$$\Rightarrow \frac{3}{4} \left(\frac{16}{3} - \frac{16}{4}\right)$$

$$\Rightarrow \frac{12}{1}\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$\Rightarrow 1$$

$\therefore \boxed{\mu = 1}$

$\Rightarrow$ Variance $(\sigma^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - \mu^2$

$$\Rightarrow \int_{0}^{2} x^2 \cdot \frac{3}{4} x(2-x)\,dx - 1$$

$$\Rightarrow \frac{3}{4} \int_{0}^{2} (2x^3 - x^4) - 1$$

$$\Rightarrow \frac{3}{4} \left(2\frac{x^4}{4} - \frac{x^5}{5}\right)_0^2 - 1$$

$$\Rightarrow \frac{3}{4}\left(\frac{2 \times 16}{4} - \frac{32}{5}\right) - 1$$

$$\Rightarrow 24\left(\frac{1}{20}\right) - 1$$

$$\Rightarrow 6/5 - 1$$

$$= 1/5$$

$\therefore \boxed{\sigma^2 = 1/5}$

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$$\Rightarrow P(1 \le x \le 2) = \frac{3}{4}\int_1^2 x(2-x)\,dx$$

$(x^2)_1^2 - \left(\frac{x^3}{3}\right)_1^2$

$4-1-\left(\frac{8}{3}-\frac{1}{3}\right)$

$$= \frac{3}{4}\left(2\frac{x^2}{2} - \frac{x^3}{3}\right)_1^2$$

$3 - \frac{7}{3}$. $2/3 - 1$

$-1/3$

$$\Rightarrow \frac{3}{4}\left(4 - \frac{8}{3} - \left(1 - \frac{1}{3}\right)\right)$$

$$\Rightarrow \frac{3}{4}\left(4 - \frac{8}{3} - \frac{2}{3}\right) \quad 4 - \left(\frac{10}{3}\right)$$

$\frac{12}{3}$

$$\Rightarrow \frac{3}{4}(2)$$

$$= 3/2$$

$$= \cancel{1/2} \quad 2/3 \times \frac{3}{4}$$

$$P(1 \le x \le 2) = \underline{\underline{1/2}}$$

Q: If X is a continuous R.V and $y = aX+b$ proove that $E(Y) = aE(X)+b$ and $V(Y) = a^2V(X)$, where 'V' stands for varience & a, b are con-stants.

Sol: given $Y = aX+b$

$$E(Y) = E(aX+b)$$

According to defination of Expectation

$$E(Y) = \int_{-\infty}^{\infty} (aX+b)\cdot f(x)\,dx$$

$$= a\int_{-\infty}^{\infty} x f(x)\,dx + b\int_{-\infty}^{\infty} f(x)\,dx$$

$$= a\cdot E(X) + b(1)$$

$$= \underline{aE(X)+b}$$

$\rightarrow \underline{V(Y) = a^2 \cdot V(X)}$

$\Rightarrow V(Y) = \int_{-\alpha}^{\alpha} (ax+b)^2 \cdot f(x)\,dx$

$= \int_{-\alpha}^{\alpha} (a^2x^2 + b^2 + 2abx)\, f(x)\,dx$

$= a^2 \int_{-\alpha}^{\alpha} x^2 f(x)\,dx + b^2 \int_{-\alpha}^{\alpha} f(x)\,dx + 2ab\int_{-\alpha}^{\alpha} x f(x)\,dx$

$= a^2 V(x) + b^2 + 2ab \cdot E(x).$

---

* Given $y = ax+b$ —— (1)

then $E(y) = E(ax+b)$

$E(Y) = aE(x) + b$

$E(Y) = aE(X) + Y - aX$ $\quad (\because$ from (1)$)$

$Y - E(Y) = aX - aE(X)$

$Y - E(Y) = a(X - E(X))$ —— (2)

squaring and taking expectation of both sides in (2)

$E(Y - E(Y))^2 = a^2 E(X - E(X))^2$ $\left\{ \text{it is for discrete r.v.} \right\}$

$\boxed{V(Y) = a^2 V(X)}$

---

* $V(Y) = \int_{-\alpha}^{\alpha} (Y - E(Y))^2 f(x)\,dx = \int_{-\alpha}^{\alpha} [aX + b - E(aX+b)]^2 f(x)\,dx$

$= \int_{-\alpha}^{\alpha} [aX + \cancel{b} - a \cdot E(X) - \cancel{b}]^2 f(x)\,dx$

$= \int_{-\alpha}^{\alpha} a^2 [X - E(X)]^2 \cdot f(x)\,dx$

$V(Y) = a^2 \cdot \int_{-\alpha}^{\alpha} [X - E(X)]^2 f(x)\,dx$

$\boxed{V(Y) = a^2 \cdot V(X)}$ $\left\{ \text{It is for Continuous R.V.} \right\}$

Q: The trouble shooting of an IC chip in a circuit in ~~a circuit~~ in a R.V X whose distribution is given by

$$F(x) = \begin{cases} 0, & x \leq 3 \\ 1 - 9/x^2, & x > 3. \end{cases}$$ where 'x' denotes

the no. of a years. Find the probability that the IC chip will work properly

(i) < 8 years

(ii) beyond 8 years

(iii) b/w 5 to 7 years

(iv) Anywhere from 2 to 5 years.

Sol: From Cumilative distribution Function

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

(i) < 8 years ; i.e., $x < 8$

$$\therefore P(X \leq 8) = \int_{-\infty}^{3} f(x)\,dx + \int_{3}^{8} f(x)\,dx$$

$$= 0 + \int_{3}^{8} f(x)\,dx.$$

$$\therefore P(X \leq 8) = \int_{-\infty}^{8} f(x)\,dx$$

$\boxed{F(x) = \int f(x)\,dx + C}$

$$= \int_{-\infty}^{3} f(x)\,dx + \int_{3}^{8} f(x)\,dx$$

$$= 0 + \int_{3}^{8} \left(1 - 9/x^2\right) dx$$

$$= (x)_3^8 + 9\left(\frac{1}{x}\right)_6$$

$$\Rightarrow \frac{64-9}{64} \Rightarrow \frac{55}{64}$$

$1/x$

$(-1/x^2)$

$\frac{x^2(0) - 2x}{x^4}$

$-\frac{2x}{x^3}$

$-1/x^2$

(ii) $P(X>8) = 1 - P(X \leq 8)$

$$= 1 - \frac{55}{64}$$

$$= 9/64$$

64
55
9

(iii) $P(5 \leq X \leq 7) = \int_5^7 f(x)\,dx = F(7) - F(5)$

$$= 1 - 9/7^2 - (1 - 9/5^2)$$

$$= \cancel{1} - 9/7^2 - \cancel{1} + 9/5^2$$

$$= \frac{9}{25} - \frac{9}{49}$$

$$= 9\left(\frac{49-25}{25\times 49}\right)$$

$$= \frac{216}{1225} \,//$$

(iv) $P(2 \leq X \leq 5) = \int_2^5 f(x)\,dx = F(5) - F(2)$

$$= 1 - \frac{9}{5^2} - \left(1 - \frac{9}{2^2}\right)$$

$$= \frac{9}{4} - \frac{9}{25}$$

$$= 9\left(\frac{21}{4\times 25}\right)$$

$$= \frac{189}{200}$$

Moments :

The term moment is generally used in physics and it provides a measure of turning or rotating the effect of a force about some point.

→ Moment is defined as A.M of various powers of deviation of the item from their mean. {assumed or actual mean} will give the required power of moment of the distribution.

→ If the deviations of the items are taken from the A.M of the distribution it is known Central moment.

Central moments (or) Moments about actual mean

(i) For individual Series :

moment 1 $\mu_1 = \frac{\sum x_i}{n}$ where $\boxed{x_i = X - \bar{x}}$

moment 2 $\mu_2 = \frac{\sum x^2}{n}$

⋮

r^{th} moment $\mu_r = \frac{\sum x^r}{n}$

(ii) Frequency distribution :

moment 1 → $\mu_1 = \frac{\sum f_i x_i}{N}$ where $x_i = X - x$

moment 2 $\mu_2 = \frac{\sum f_i x_i^2}{N}$ . . . . . $\mu_r = \frac{\sum f_i x_i^r}{N}$.

* properties of Central moment

(i) the first moment about mean is always '0' $\boxed{\mu_1 = 0}$

(ii) the 2nd moment about mean measures

$$\mu_2 = \sigma^2 \Rightarrow \sigma = \sqrt{\mu_2}$$

(iii) the 3rd moment about mean measures "skewness" of a given distribution

⇒ If $\mu_3 > 0$; the distribution is positively skewed

⇒ If $\mu_3 < 0$; the distribution is negatively skewed

⇒ If $\mu_3 = 0$; the distribution is symmetrical

(iv) the 4th moment about mean measures "kurtosis" of a frequency distribution

(v) Skewness and kurtosis of a distribution are calculated from μ_2, μ_3 and μ_4

* Skewness = $\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3}}$ —— ①

* Kurtosis = $\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}}$ —— ②

From ① and ②

* $\sqrt{\beta_1} = \pm \frac{\mu_3}{\mu_2^{3/2}}$

→ the sign of β_1 depends on sign of 'μ_3'

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→ **Raw Moments (or) Moments about Orbitary Origins.**

→ when the actual mean of a distribution is "fraction value", in such a case we calculate the mean by an orbitarary Origin 'A', for calculation of moments and then convert these moments into the moments about the actual mean

→ Moments about the orbitarary moment is called "Raw Moments". (μ_r') $\{\mu_r'\}$

→ the r^{th} moment about point A is given by $\mu_r' = \frac{\sum f_i d_i^r}{N}$; where : $d_i = x_i - A$

→ In Case of class intervel Frequency Distrubution

$$\mu_r' = \frac{\sum f_i (d_i')^r}{N} \; ; \text{ where } d_i' = \frac{x_i - A}{C}$$

$\boxed{C = \text{length of class}}$

→ Relation b/w moments about mean in terms of mean about any points and vice-versa

① (i) $\mu_1 = 0$

(ii) $\mu_2 = \mu_2' - (\mu_1')^2$

(iii) $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

(iv) $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

②

(i) $\mu_1' = \bar{x} - A$

(ii) $\mu_2' = \mu_2 + (\mu_1')^2$

(iii) $\mu_3' = \mu_3 + 3\mu_2\mu_1' + (\mu_1')^3$

(iv). $\mu_4' = \mu_4 + 4\mu_3\mu_1 + 6\mu_2(\mu_1')^2 + (\mu_1')^4$.

→ Coefficient of Skewness:

⇒ The square root of skewness that is,
$\sqrt{\beta_1} = \frac{(\pm\mu_3)}{(\mu_2)^{3/2}}$ is called Coefficient of Skewness.

⇒ The sign of the Skewness is determined by the sign of 'μ_3'.

problem:

① Find the first four moments for the set of no. 2, 4, 6, 8

Given data = (2, 4, 6, 8)

$\mu = \bar{x} = \frac{2+4+6+8}{4} = 5$

(i) $\mu_1 = \frac{\sum x_i}{n} = 0$ where $x_i = x - \bar{x}$

(ii) $\mu_2 = \frac{\sum x_i^2}{n} = \frac{20}{4} = 5$

(iii) $\mu_3 = \frac{\sum x_i^3}{n} = \frac{0}{4} = 0$

(iv) $\mu_4 = \frac{\sum x_i^4}{n} = \frac{164}{4} = 36$

x_i	$x_i = x-5$	x_i^2	x_i^3	x_i^4
2	−3	9	−27	81
4	−1	1	−1	1
6	1	1	1	1
8	3	9	27	81
	0	20	0	164

$\sum x_i^2 = 20$

$\sum x_i^3 = 0$ $\quad \sum x_i = 0$

$\sum x_i^4 = 164$

(3) Calculate the first 4 moments of the following distribution about the mean, also evaluate β_1 and β_2

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

Sol;

x	f_i	$d_i = x - A$	$f_i d_i$	$f_i d_i^2$	$f_i d_i^3$	$f_i d_i^4$
0	1	-4	-4	16	-64	256
1	8	-3	-24	72	-216	648
2	28	-2	-56	112	-224	448
3	56	-1	-56	56	-56	56
4	70	0	0	0	0	0
5	56	1	56	56	56	56
6	28	2	56	112	224	448
7	8	3	24	72	216	648
8	1	4	4	16	64	256
	256		$\Sigma f_i d_i = 0$	$\Sigma f_i d_i^2 = 512$	$\Sigma f_i d_i^3 = 0$	$\Sigma f_i d_i^4 = 2816$

$N = \frac{\Sigma f_i}{\cancel{N}} = \frac{256}{\cancel{1}}$ $\boxed{N = 256}$

→ Moments about point $x = 4$ are

$\mu_1' = \frac{\Sigma f_i d_i}{N} = \frac{0}{256} = 0$; $\mu_2' = \frac{\Sigma f_i d_i^2}{N} = \frac{512}{256} = 2$

$\mu_3' = \frac{\Sigma f_i d_i^3}{N} = \frac{0}{256} = 0$; $\mu_4' = \frac{\Sigma f_i d_i^4}{N} = \frac{2816}{256} = 11$

→ Moments about mean are :

$\mu_1 = 0$; $\mu_2 = \mu_2' - (\mu_1')^2 = 2 - 0 = 2$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 = 0 - 3(2)(0) + 2(0)^3 = 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4 = 11$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0$; $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{11}{4} = 2.75$ //

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→ Moment generating function

→ This is a tool used to calculate the higher moments

def: The moment generating funⁿ of a R.V 'x' about the origin whose probability density function $f_x(x)$ is given by.

$$M_x(t) = E(e^{tx}) = \begin{cases} \sum e^{tx} p(x) \text{ ; Descrete R.V} \\ \int_{-\infty}^{\infty} e^{tx} f_x(x) \text{ ; Continuous R.V.} \end{cases}$$

→ Since $M_x(t)$ is used to generate moments, so it is known as moment generating funⁿ

(i) $$\mu_r' = \left(\frac{d^r}{dt^r}\left[M_x(t)\right]\right)_{t=0}$$

Here "μ_r'" is the rth moment of R.v 'x' about the origin

(ii) The moment generating fun^n of a R.V 'x' about the point $x=a$ is defined as .

$M_x(t)$ (about $x=a$)

$$= \int_{-\infty}^{\infty} e^{t(x-a)} \cdot f(x)\,dx.$$

Properties:

① let ~~Y~~ $Y = aX+b$ where 'x' is R.V with moment generating Fun^n $M_X(t)$, then

$\cancel{M_X(Y) = a\,M_X(X) + b.}$

$$M_X(Y) = M_X(aX+b)$$
$$= e^{bt} M_X(at).$$

② $M_{KX}(t) = M_X(Kt)$, where 'K' is a ± constant.

③ If X, Y are two independent R.V having the M.G.F $M_X(t)$ and $M_Y(t)$ then the moment.g. Fun of $(X+Y)$ is

$$M_{X+Y}(t) = M_X(t) \cdot M_Y(t).$$

④ A R.V 'x' may have no moments even if its moment generating function exists

⑤ A R.V 'x' can have all or Some moments but M.G.F doesn't exist perhaps at one point.